Radiation Safety & Control Services, Inc.

Introduction to Uncertainty Calculations in Instrument Calibrations

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Professional Enrichment Program Continuing Education Lecture

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Terminal Objective

To understand the basic concepts of determining the total uncertainty in exposure rate for instruments calibrated using gamma-emitting sources

Enabling Objectives

- Define the concept of accuracy and precision
- List some major factors which affect instrument accuracy
- Identify both systematic and random errors associated with instrument calibrations
- Understand the method for calculating instrument response and its associated error
- Understand how the accuracy criteria is applied to the calibration process

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Outline

- General Calibration Process
- Uncertainty Parameters
- Determining Source Exposure Rate Uncertainty
- Determining Random and Systematic Uncertainty in Detector Measurements
- Determining Total Uncertainty in Measurement Results

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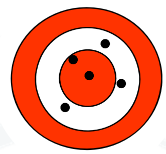
Accuracy and Precision

- **Accuracy:** a measure of how well a true quantity is estimated, measured value/true value.



- **Precision:** a measure of reproducibility

Instrument Calibrations

- Calibration goal is to provide a calibration that will yield an "acceptably accurate" estimate of the desired quantity (i.e. exposure or dose rate) when used in the field
- To determine that a calibration is "acceptably accurate" one must know the uncertainty in the measurement

Types of Uncertainty

- **Random**: Uncertainties in the random variations in the measurement process that quantifies the precision
- **Systematic**: Uncertainties that cannot be estimated by statistical methods

Random Uncertainties

Random uncertainties can be calculated using the following equation:

$$\delta_{Random} = t * \sigma_R$$

Where:

t = Student t value for particular degrees of freedom to yield a given probability that the true value X will be included in the confidence interval

σ_R = Standard Deviation in the value with random error

Value of Student t-factor

Degrees of Freedom, n-1	Probability 0.50	0.90	0.95	0.99
1	1.000	6.31	12.71	63.7
2	0.816	2.92	4.30	9.92
3	0.765	2.35	3.18	5.84
4	0.741	2.13	2.78	4.60
5	0.727	2.02	2.57	4.03
6	0.718	1.94	2.45	3.71
7	0.711	1.90	2.36	3.50
9	0.703	1.83	2.26	3.25
14	0.692	1.76	2.14	2.98
19	0.688	1.73	2.09	2.86
29	0.683	1.70	2.04	2.76
49	0.679	1.68	2.01	2.68
99	0.677	1.66	1.98	2.63
∞	0.674	1.64	1.96	2.58

Reference: NCRP 112, Table 2.2

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Systematic Uncertainties

- May result from a number of causes
  - Errors in reading instrument response
  - Source to detector distance errors
  - Attenuator placement errors
- May be positive or negative
- May not be normally distributed
- Should try to be eliminated by investigation and correction

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Systematic Uncertainties

- Systematic uncertainties can be estimated by determining the apparent standard deviation, *u*
- The apparent standard deviation, *u*, can be estimated as 1/3 of the maximum systematic uncertainty
- For 95% confidence, 2*u* is the range of uncertainty around the mean

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Overall Uncertainty

- Systematic uncertainty standard deviation $$u = \frac{\delta \, \mathrm{SysMax}}{3}$$
- Random uncertainty standard deviation $$\sigma_R$$
- Overall Uncertainty $$\pm \left[(t\sigma_R)^2 + (ku)^2 \right]^{\frac{1}{2}}$$
  - The value of k at the 95% confidence level is 2

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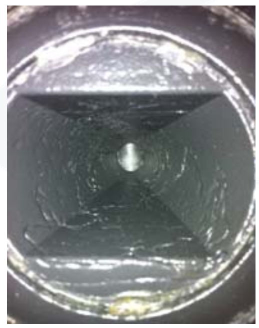
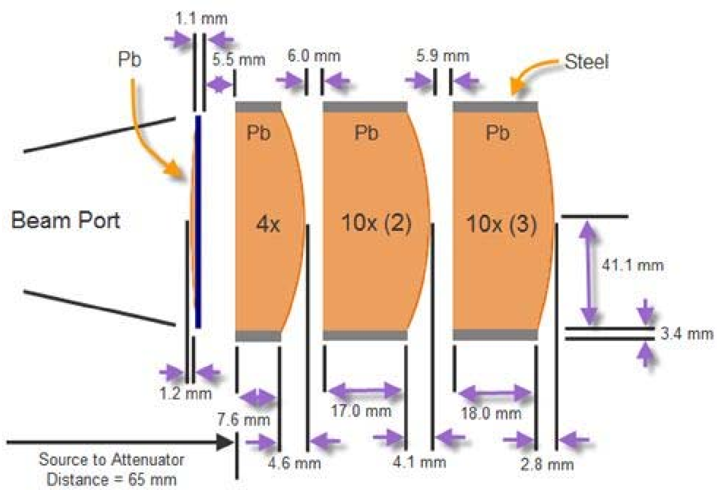
Case Study in Determining Exposure Rate Calibration Uncertainty



Assumptions Used

- Cs-137 Calibration Beam Source installed on track system
- 3 attenuator used in multiple combinations
- Multiple instruments calibrated
  – Ion Chambers
  – GM Detectors
  – Scintillation Detectors
  – Solid State Detectors



Uncertainty Evaluation

- Evaluate the “true” exposure rate and potential uncertainties in the “true” values assumed
- Evaluate the potential uncertainties in various parameters associated with calibration of each type of instrument
  - Random
  - Systematic

Evaluate the "true" Exposure Rate and its Uncertainty

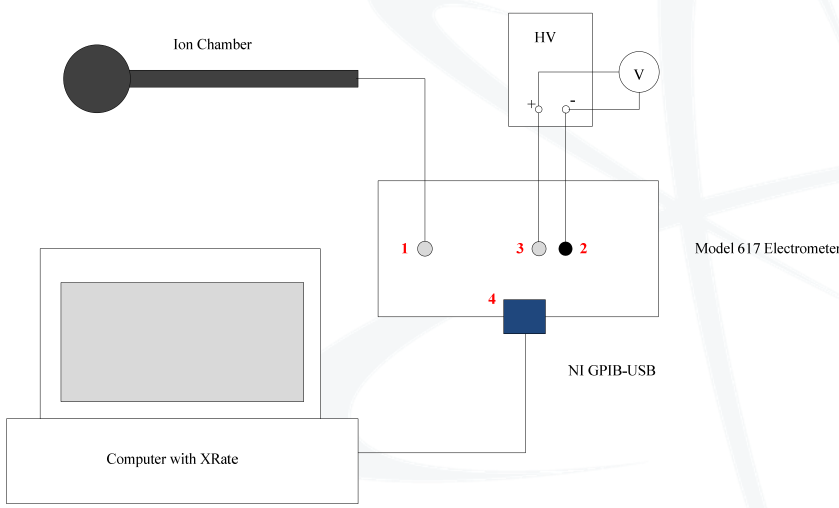
- Use a transfer standard instrument such as an Exradin Shonka – Wychoff ionization chamber[1] or HPIC for low dose rates
  - [1]Used with a calibrated electrometer system to measure current
- Transfer Standards calibrated and traceable to the National Institute of Standards

Uncertainty Evaluation

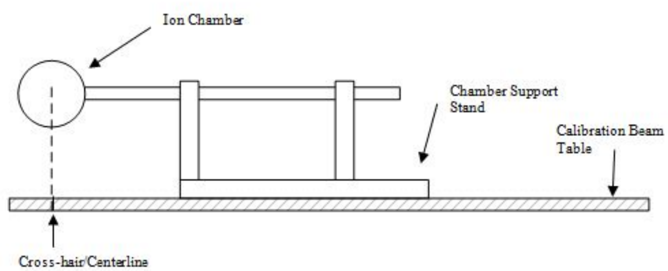
- Review of measurement process identified both random and systematic errors associated with determination of true exposure rates
- Radom Error
  - Associated with the statistical results of the transfer instruments
- Systematic Errors
  - Detector Placement (distance)
  - Ion Chamber Calibration
  - Electrometer Calibration
  - Temperature
  - Pressure
  - Exposure rate curve fit (residuals)

Evaluation of "true" Exposure Rate

- Background measurements recorded using a lead shield in front of the source
- Unattenuated and attenuated measurements taken at source to center of detector at distances of 30, 50, 80, 100, 200, 300 cm
- Six measurements taken at each locations and averaged to yield exposure rate at each location



Reference Chamber Support



Exposure Rate Curve Development

- Exposure rates plotted and a power trend line created to get the base and exponential values of the experimental measurements
- Power equation fit yields correlation coefficients $\dot{X} = a * x^b$
- Percent residuals determined between the fit functions and the measured data

Exposure Rate Power Functions

Beam			
Source	Base	Exponent	Corr. Fit
Attenuator	*a*	*b*	r^2
1x	6.279100E+02	-2.004660E+00	9.999627E-01
4x	1.345762E+02	-1.973129E+00	9.999258E-01
10x2	6.621852E+01	-2.016499E+00	9.999716E-01
10x3	6.417209E+01	-1.999544E+00	9.999254E-01
40x2	2.215078E+01	-2.060330E+00	9.996309E-01
40x3	1.783889E+01	-2.008843E+00	9.993982E-01
100x	9.883114E+00	-2.059398E+00	9.986914E-01
400x	1.605974E+00	-1.938399E+00	9.994158E-01

$$\dot{X} = a * x^b$$

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Table of Residuals

Distance (cm)	30 (A6)	50 (A6)	80 (A6)	100 (A6)	100 (PIC)	200 (PIC)	300 (PIC)
1x	-0.268%	1.192%	-0.382%	-0.250%	-0.124%	-1.440%	1.300%
4x	0.795%	-1.250%			1.201%	-2.149%	1.455%
10x2	-0.024%				-0.508%	1.528%	-0.979%
10x3	1.090%				-2.564%	0.889%	0.631%
40x2					0.943%	-2.511%	1.617%
40x3					1.161%	-3.079%	1.992%
100x					1.761%	-4.618%	3.029%
400x					1.103%	-2.929%	1.893%

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Overall Random Uncertainty

$$\delta_{Random} = t * \sigma_R$$

$$\sigma_R = R\sqrt{\left(\frac{\sigma_I}{I}\right)^2}$$

σ_R = Standard deviation in cal value
σ_I = Standard deviation in the mean instrument response
T = True calibration value
I = Mean of instrument response
R $= \frac{I}{T} = 1$

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Overall Random Uncertainty

$$\sigma_R = \left(\frac{\sigma_{\bar{I}}}{\bar{I}}\right) = \frac{\frac{\sqrt{\sum_{i=1}^{N}\frac{(I_i - \bar{I})^2}{(N-1)}}}{\sqrt{N}}}{\bar{I}}$$

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Overall Random Uncertainty

- Several measurements using various attenuators and distance used to compute relative errors
- Maximum relative error used in overall random uncertainty calculation

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Random Uncertainties Associated with Beam Measurements

	100 cm				200 cm				300 cm			
Attenuation	# Trials	Rel. Error	t-value	Uncertainty	# Trials	Rel. Error	t-value	Uncertainty	# Trials	Rel. Error	t-value	Uncertainty
1x	6	0.0014	2.4469	0.0034	6	0.0008	2.4469	0.0020	6	0.0009	2.4469	0.0021
4x	6	0.0012	2.4469	0.0028	6	0.0021	2.4469	0.0050	6	0.0019	2.4469	0.0046
10x2	6	0.0008	2.4469	0.0019	6	0.0014	2.4469	0.0033	6	0.0014	2.4469	0.0035
10x3	5	0.0011	2.5706	0.0028	6	0.0035	2.4469	0.0086	6	0.0021	2.4469	0.0051
40x2	6	0.0025	2.4469	0.0060	6	0.0024	2.4469	0.0058	6	0.0036	2.4469	0.0088
40x3	6	0.0075	2.4469	0.0184	6	0.0014	2.4469	0.0034	7	0.0030	2.3646	0.0071
100x	2	0.0040	4.3027	0.0174	2	0.0011	4.3027	0.0047	6	0.0045	2.4469	0.0110
400x	6	0.0036	2.4469	0.0088	7	0.0080	2.3646	0.0190	6	0.0082	2.4469	0.0200
Maximum Uncertainty			0.0200									

Note: 0.0082 (0.82%) was the largest observed relative error

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Overall Random Uncertainty

$$\delta_{Random} = t * \sigma_R$$

$$\delta_{Random} = 2.447 * 0.0082$$

$$\delta_{Random} = 2.0\%$$

Systematic Uncertainties

$\delta_{distance\ max}$ = maximum error associated with distance = 3.9%

$\delta_{distance}$ = 2 sigma (95% confidence) error associated with distance = $2\left(\frac{3.9\%}{3}\right) = 2.6\%$

$\delta_{ion\ chamber}$ = 2 sigma (95% confidence) error associated with ion chamber calibration = 1.6%

$\delta_{Electrometer}$ = 2 sigma (95% confidence) error associated electrometer calibration = 0.20%

Systematic Uncertainties

$\delta_{temperature}$ = 2 sigma (95% confidence) error associated with temperature monitoring device calibration (+/- 1.0 C) = 0.34%

$\delta_{pressure}$ = 2 sigma (95% confidence) error associated with pressure monitoring device calibration = 1.0%

$\delta_{residuals\ max}$ = maximum error associated with curve fit residuals = 4.4%

$\delta_{residuals}$ = 2 sigma (95% confidence) error associated with curve fit residuals = $2\left(\frac{4.4\%}{3}\right) = 2.9\%$

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Overall Systematic Uncertainty

$$\pm_{systematic} = \sqrt{\left(k\frac{\delta_{distance\ max}}{3}\right)^2 + (\delta_{ion\ chamber})^2 + (\delta_{electrometer})^2 + (\delta_{temperature})^2 + (\delta_{pressure})^2 + \left(k\frac{\delta_{residuals\ max}}{3}\right)^2}$$

= 4.3%

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Overall Total Uncertainty in Exposure Rate

$$\pm_{total} = \sqrt{(\pm_{random})^2 + (\pm_{systematic})^2} = 4.8\ \%$$

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Total Uncertainty in Instrument Calibration

- Requires knowledge of "true" beam exposure uncertainty and meter uncertainties
  - Meter distance placement
  - Meter reading
  - Pressure and temperature (for unsealed detectors only)
- Calibration uncertainty is a combination of exposure uncertainty and meter uncertainties

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Meter Uncertainties

- Meter placement
  - Can be estimated to be the same as the ion chamber distance uncertainty
- Meter reading
  - Can be ascertained experimentally by taking a series of meter reading by different individuals and calculating the standard deviation
  - Can be ascertained by assuming a maximum error in meter reading
- Pressure and Temperature
  - Can be assumed to be the same uncertainties as used to calculate total exposure uncertainty
- Other uncertainties related to calibration specific calibration conditions

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Meter Reading Uncertainties

	Distance	Measurer 1	Measurer 2	Measurer 3
1 R 44-9	16.6	16.4	16.5	16.7
1 R 44-6	16.6	16.4	17	16.5
1 R ND2000	16.6	17	16.5	17.2
1 R DMC-2000	16.6	16.8	16.6	16.9
200 mR 44-9	37.1	36.8	37.3	36.8
200 mR 44-6	37.1	37	37.2	37.1
200 mR ND2000	37.1	37.2	37	37.8
200 mR DMC-2000	37.1	36.8	37.1	37.3
40 mR 44-9	82.9	82.9	82.8	83
40 mR 44-6	82.9	82.1	82.2	82.7
40 mR ND2000	82.9	83	82.5	83
40 mR DMC-2000	82.9	82.8	82.7	83.1

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Meter Reading Uncertainties

- The average error for all measurements is 0.75%
- The maximum error for an individual measurement (at the 16.6 cm distance) = 3.61%
- The standard deviation in meter reading can be estimated as $\sigma_{Meter} = \frac{\delta_{Meter\ Max}}{3} = \frac{3.61}{3}$
- The 95% probability in meter read uncertainty can be calculated as $2\sigma_{Meter} = 2.4\%$

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Total Calibration Uncertainty

$$\sqrt{\delta^2_{\text{total exposure}} + \delta^2_{\text{meter reading}} + \delta^2_{\text{temp}} + \delta^2_{\text{pressure}} + \delta^2_{\text{distance}}}$$

Note: For sealed or non-gas-filled detectors, the temperature and pressure errors can be negligible

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Total Calibration Uncertainty

$\delta_{Exposure}$ = 2 sigma (95% confidence) uncertainty in exposure = 4.3% (from previous analysis)

δ_{Meter} = 2 sigma (95% confidence) uncertainty in meter reading = 2.4% (from previous analysis)

$\delta_{Temperature}$ = 2 sigma (95% confidence) error associated with temperature monitoring device calibration (+/- 1.0 C) = 0.34%

$\delta_{Pressure}$ = 2 sigma (95% confidence) error associated with pressure monitoring device calibration = 1.0%

$\delta_{Distance}$ = 2 sigma (95% confidence) error associated with distance = 2.6%

Total Calibration Uncertainty

$$\pm\sqrt{(0.043)^2 + (0.024)^2 + (0.0034)^2 + (0.01)^2 + (0.026)^2}$$

$$\pm 5.7\%$$